US – 2938

I Semester B.A./B.Sc. Examination, December 2006
(Semester Scheme)
MATHEMATICS (Paper – I)

Time: 3 Hours Max. Marks: 90

Instructions: *1) Answer **all** questions.*
*2) Answers should be written **completely** either in **English** or in **Kannada**.*

I. Answer **any fifteen** questions: **(15×2=30)**

1) Write the truth set of the proposition $p(x): x^2 - 6x + 5 = 0,\ R[p(x)] = Z$.

2) Write the negation of : $(\forall x)(\exists y) x + y = xy$.

3) Find the equivalence relation associated with the partition $P = \{\{1, 2\}, \{3, 4\}\}$ of the set $A = \{1, 2, 3, 4\}$.

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4) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 5\ \forall x \in \mathbb{R}$ is a bijection.

5) Find the n^{th} derivative of $\frac{1}{(2x+5)^5}$.

6) Find the n^{th} derivative of $\cos^3 4x$.

7) If $u = x^y$ find $\frac{\partial^2 u}{\partial x \partial y}$.

8) If $u = x^3 + y^3$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$.

9) If $u = x^2 + 3xy + y^2$ where $x = 2t$ and $y = t^2$ find $\frac{du}{dt}$.

10) If $x = u(1 - v)$, $y = uv$ find $\frac{\partial(x, y)}{\partial(u, v)}$.

11) Find $\int_0^{\pi/2} \text{Sin}^{12}x\, dx$.

12) Using the reduction formula for $\int \tan^n x\, dx$, find $\int \tan^5 x\, dx$.

13) The centroid of a triangle ABC is (2, 1, – 1). If A ≡ (1, 2, – 1) and B ≡ (2, 0, 3) find the vertex C.

14) If $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines of a line, show that $\cos^2\alpha = \text{Sin}^2\beta + \text{Sin}^2\gamma - 1$.

15) Find the direction ratios of a line perpendicular to AB and CD where A = (2, 1, 2), B = (3, 2, 0), C = (3, – 4, 1) and D = (2, 1, – 3).

16) Find the equation of the plane passing through the point (3, – 2, – 1) and parallel to the vectors $\hat{i} - 2\hat{j} + 4\hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$.

17) Find the equation of the line passing through the point (1, – 1, 2) and parallel to the line $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-2}{2}$ in the Cartesian form.

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18) Find the equation of the sphere whose centre is (– 1, 3, 2) and radius 3.

19) Find the equation of the right circular cone whose vertex is the origin, semi vertical angle 30° and axis is z axis.

20) Write the standard form of the equation of the ellipsoid and mention any two properties.

II. Answer **any two** questions: **(2×5=10)**

1) With usual notations, prove that $T[p(x) \vee q(x)] = T[p(x)] \cup T[q(x)]$.

2) Prove the following by indirect proof: "If a + b is odd and a is even then b is odd ".

3) If R and R′ are two transitive relations defined on a set A then prove that $R \cap R'$ is transitive but $R \cup R'$ need not be transitive.

4) Prove that the composition of mappings which are bijective is again a bijective

III. Answer **any three** questions: **(3×5=15)**

1) Find the n^{th} derivative of $e^{2x}\cos^2 x \cos 2x$.

2) State and prove Leibnitz's theorem for the n^{th} derivative of a product of two functions.

3) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, show that

$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = -4(x+y)^{-2}.$$

4) If $z = f(x + ay) + \phi(x - ay)$, show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

5) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$ find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

IV. Answer **any two** questions: **(2×5=10)**

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1) If $I_n = \int_0^{\pi/2} x^n \cos x \, dx \ (n > 1)$, prove that $I_n + n(n-1)I_{n-2} = \left(\frac{\pi}{2}\right)^n$.

2) Evaluate $\int_1^{\sqrt{2}} \frac{x^3}{\sqrt{x^2 - 1}} dx$.

3) Using Leibnitz's Rule, evaluate $\int_0^{\infty} e^{-x} \frac{\sin \alpha x}{x} dx$.

. Answer **any three** questions : **(3×5=15)**

1) Show that the two lines whose direction cosines satisfy the equations $l + 2m + 3n = 0$ and $mn - 4nl + 3lm = 0$ are at right angles.

2) Derive the equation of a plane in the normal form. Express it both in the vector form and Cartesian form.

3) Show that the points (1, –1, 1), (0, – 4, 1), (4, 0, 2) and (–1, 1, 0) are coplanar. Find the equation of the plane containing them.

4) Find the equation of the line through the point (2, 3, 1) and perpendicular to the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

5) Find the length and equation of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

VI. Answer **any two** questions: **(2×5=10)**

1) Find the equation of the sphere passing through the points (1, 1, 1), (1, 2, 1), (1, 1, 2) (2, 1, 1).

2) Derive the equation of a right circular cone in the form $x^2 + y^2 = z^2 \tan \alpha$.

3) Find the equation of the right circular cylinder having the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ as its axis and the line $\frac{x-4}{3} = \frac{y-3}{4} = \frac{z-2}{-5}$ as a generator.

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ಕನ್ನಡ ರೂಪಾಂತರ

I. ಯಾವುದಾದರೂ **ಹದಿನೈದು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿ: **(15×2=30)**

1) ಈ ವಾಕ್ಯದ ನಿಜಗಣವನ್ನು ಬರೆಯಿರಿ.

$$p(x): x^2 - 6x + 5 = 0,\ R[p(x)] = Z$$

2) ಇದರ ನಕಾರವನ್ನು ಬರೆಯಿರಿ.

$$(\forall x)(\exists y),\ x + y = xy$$

3) A = [1, 2, 3, 4] ಗಣದ ವರ್ಗೀಕರಣ $P = \{\{1, 2\}\ \{3, 4\}\}$ ಆಗಿದ್ದರೆ, ಅದರ ಸಮಾಂಗೀಯ ಸಂಬಂಧವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

4) $f: \mathbb{R} \to \mathbb{R}$ ಉತ್ಪನ್ನವು $f(x) = 3x + 5\ \forall x \in \mathbb{R}$ ಎಂದು ವ್ಯಾಖ್ಯಿಸಿದೆ. ಇದನ್ನು ಬೈಜೆಕ್ಷನ್ ಎಂದು ತೋರಿಸಿ.