SO – 219

V Semester B.Sc./B.A. Examination, October/November 2011
(Semester Scheme)
MATHEMATICS (Paper – V)

Time : 3 Hours Max. Marks : 90

Instruction : Answer all questions.

I. Answer any fifteen questions : (15×2=30)

1. In a Ring (R, +, .), prove that (–a) (–b) = ab for all a, b ∈ R.

2. Give an example of

   a) a ring with zero divisions.

   b) a ring without zero divisions.

3. Define left ideal and right ideal of a Ring R.

4. Define a Field. Give an example.

5. Prove that the intersection of two subrings of a ring is a subring.

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6. If R is a ring and a∈ R such that $r(a) = \{x \in R \mid ax = 0\}$ is a right ideal.

7. If $\vec{r} = t\hat{i} - t^2\hat{j} + \sin t.\hat{k}$ find $\frac{d\vec{r}}{dt}$ and $\frac{d^2\vec{r}}{dt^2}$ at t = 0.

8. If $\vec{r} = (\cos \omega t)\hat{i} + (\sin \omega t)\hat{j}$ show that $\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$.

9. Show that a necessary condition for a curve to lie on a plane is that $\tau = 0$ at all points.

10. For a curve x = t, y = t^2 and z = $\frac{2t^3}{3}$ find the unit tangent vector at t = 1.

11. Find spherical co-ordinates of a point whose Cartesian co-ordinates are $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}, -1\right)$.

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12. If $\phi(x, y, z) = xy^2 + yz^3$ find $\nabla\Phi$ at $(2, -1, 1)$.

13. Find the constant a so that $\vec{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x-az)\hat{k}$ solenoi

14. Prove that $\text{div}\,(\text{Curl}\,\vec{F}) = 0$.

15. Show that $\nabla^2\left(\frac{1}{r}\right) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

16. Show that $\vec{F} = (\sin y + z)\hat{i} + (x\cos y - z)j + (x-y)k$ is irrotational.

17. Show that $2x - 3x^3 = \frac{1}{5}P_1(x) - \frac{6}{5}P_3(x)$.

18. Show that $P_n^1(1) = \frac{n(n+1)}{2}$.

19. Show that $J_{-n}(x) = (-1)^n J_n(x)$.

20. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

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II. Answer **any four** questions :

1) Prove that R = {0, 1, 2, 3, 4} is a ring under addition modulo 5 and multiplication modulo 5.

2) Define the centre of a ring and prove that centre of a ring is a subring of R.

3) Prove that every finite integral domain is a field.

4) Find all principal ideals of the ring R = {0, 1, 2, 3, 4, 5} with respect to addition modulo 6 and multiplication modulo 6.

5) Prove that a commutative ring with unity is a field if and only if {0} is a maximal ideal of a ring.

6) Define Kernel of a homomorphism. Prove that $f : R \to R'$ is an isomorphism if and only if Kernel $f = \{0\}$.

III. Answer **any three** questions : (3×5=15)

1) For a space curve $x = a\cos t$, $y = a\sin t$, $z = bt$ find the unit tangent $\hat{t}$ and principal normal $\hat{n}$.

2) Derive the expression for curvature and torsion for the space curve $\vec{r} = \vec{r}(u)$.

3) Find the equation of the tangent plane and normal line at (1, 2, –1) to the surface $x^2 + 2y^2 + 3z^2 = 12$.

4) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at (1, 2, –1).

5) Express the vector $\vec{F} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar co-ordinate.

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IV. Answer **any three** questions : (3×5=15)

1) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

2) Show that $\nabla \left\{\frac{f(r).\vec{r}}{r}\right\} = \frac{1}{r^2}\frac{d}{dr}\left\{r^2 f(r)\right\}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

3) If $\vec{F} = (x + y + az)\,\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ find a, b, c such that $\vec{F}$ is irrotational and then find Φ such that $\vec{F} = \nabla\Phi$.

4) If $\vec{F}$ is a vector point function prove that $\text{Curl}\left(\text{Curl}\,\vec{F}\right) = \text{grad}\,(\text{div}\vec{F}) - \nabla^2\vec{F}$.

5) Derive an expression for divergence of a vector function in orthogonal curvilinear co-ordinates.

V. Answer any two questions :

1) Show that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$ for $m \neq n$.

2) Show that $x P_n^1(x) - P_{n-1}^1(x) = n P_n(x)$.

3) Show that $\int_{0}^{1} P_{2n}(x) dx = 0$.

OR

Prove that $2n J_n(x) = x \left[J_{n+1}(x) + J_{n-1}(x) \right]$.

4) Show that $\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x) \right] = \frac{2}{x}\left[n J_n^2(x) - (n+1) J_{n+1}^2(x) \right]$.